Solutions to short-answer questions

1 a $(x^3)^4 = x^{3\times 4} = x^{12}$

b $(y^{-12})^{\frac{3}{4}} = y^{-12\times\frac{3}{4}} = y^{-9}$

c $3x^{\frac{3}{2}} \times -5x^4 = (3\times -5)x^{\frac{3}{2}+4} = -15x^{\frac{11}{2}}$

d $(x^3)^{\frac{4}{3}} \times x^{-5} = x^{3\times\frac{4}{3}} \times x^{-5} = x^{4-5} = x^{-1}$

2 $23\times 10^{-6} \times 14 \times 10^{15} = (14\times 23)\times 10^{15-6} = 322\times 10^9 = 3.22\times 10^{11}$

3 a $\frac{3x}{5} + \frac{y}{10} - \frac{2x}{5} = \frac{6x+y-4x}{10} = \frac{2x+y}{10}$

b $\frac{4}{x} - \frac{7}{y} = \frac{4y-7x}{xy}$

c $\frac{5}{x+2} + \frac{2}{x-1} = \frac{5(x-1)+2(x+2)}{(x+2)(x-1)} = \frac{5x-5+2x+4}{(x+2)(x-1)} = \frac{7x-1}{(x+2)(x-1)}$

d $\frac{3}{x+2} + \frac{4}{x+4} = \frac{3(x+4)+4(x+2)}{(x+2)(x+4)} = \frac{3x+12+4x+8}{(x+2)(x+4)} = \frac{7x+20}{(x+2)(x+4)}$

e $$\begin{aligned}\frac{5x}{x+4} + \frac{4x}{x-2} - \frac{5}{2} &= \frac{10x(x-2)+8x(x+4)-5(x+4)(x-2)}{2(x+4)(x-2)}\\ &= \frac{10x^2-20x+8x^2+32x-5(x^2+2x-8)}{2(x+4)(x-2)}\\ &= \frac{10x^2-20x+8x^2+32x-5x^2-10x+40}{2(x+4)(x-2)}\\ &= \frac{13x^2+2x+40}{2(x+4)(x-2)}\end{aligned}$$

f $$\frac{3}{x-2}-\frac{6}{(x-2)^2}=\frac{3(x-2)-6}{(x-2)^2}$$
$$=\frac{3x-6-6}{(x-2)^2}$$
$$=\frac{3x-12}{(x-2)^2}$$
$$=\frac{3(x-4)}{(x-2)^2}$$

4 a $$\frac{x+5}{2x-6}\div\frac{x^2+5x}{4x-12}=\frac{x+5}{2x-6}\times\frac{4x-12}{x^2+5x}$$
$$=\frac{x+5}{2(x-3)}\times\frac{4(x-3)}{x(x+5)}$$
$$=\frac{4}{2x}=\frac{2}{x}$$

b $$\frac{3x}{x+4}\div\frac{12x^2}{x^2-16}=\frac{3x}{x+4}\times\frac{x^2-16}{12x^2}$$
$$=\frac{3x}{x+4}\times\frac{(x-4)(x+4)}{12x^2}$$
$$=\frac{3x(x-4)}{12x^2}$$
$$=\frac{x-4}{4x}$$

c $$\frac{x^2-4}{x-3}\times\frac{3x-9}{x+2}\div\frac{9}{x+2}=\frac{x^2-4}{x-3}\times\frac{3x-9}{x+2}\times\frac{x+2}{9}$$
$$=\frac{(x-2)(x+2)}{x-3}\times\frac{3(x-3)}{x+2}\times\frac{x+2}{9}$$
$$=\frac{(x+2)(x-2)}{3}=\frac{x^2-4}{3}$$

d $$\frac{4x+20}{9x-6}\times\frac{6x^2}{x+5}\div\frac{2}{3x-2}=\frac{4(x+5)}{3(3x-2)}\times\frac{6x^2}{x+5}\times\frac{3x-2}{2}$$
$$=\frac{4\times 6x^2}{3\times 2}=4x^2$$

5 a $3\times 10^{12}\div(1.5\times 10^6)=2\times 10^6$
$2\times 10^6=2\ 000\ 000$ photos can be stored.

b $120 \text{ bits}=15 \text{ bytes//}$
$3\times 10^{12}\div(15\times 10^6)=2\times 10^5$ seconds

6 Let g be the number of games the team lost. They won $2g$ games and drew one third of 54 games, i.e. 18 games.
$$g+2g+18=54$$
$$3g=54-18$$
$$=36$$
$$g=12$$
They have lost 12 games.

7 Let b be the number of blues CDs sold. The store sold $1.1b$ classical and $1.5(b+1.1b)$ heavy metal CDs, totalling 420 CDs.
$$b+1.1\,b+1.5\times 2.1b=420$$
$$5.25b=420$$
$$b=\frac{420}{5.25}$$
$$=80$$

$$
\begin{aligned}
1.1b &= 1.1 \times 80 = 88 \\
1.5 \times 2.1b &= 1.5 \times 2.1 \times 80 \\
&= 252
\end{aligned}
$$

80 blues, 88 classical and 252 heavy metal (totalling 420)

8 **a**
$$
\begin{aligned}
V &= \pi r^2 h \\
&= \pi \times 5^2 \times 12 \\
&= 300\pi \approx 942 \text{ cm}^3
\end{aligned}
$$

b
$$
\begin{aligned}
h &= \frac{V}{\pi r^2} \\
&= \frac{585}{\pi \times 5^2} \\
&= \frac{117}{5\pi} \approx 7.4 \text{ cm}
\end{aligned}
$$

c
$$
\begin{aligned}
r^2 &= \frac{V}{\pi h} \\
r &= \sqrt{\frac{V}{\pi h}} \text{ (use positive root)} \\
&= \sqrt{\frac{786}{\pi \times 6}} \\
&= \sqrt{\frac{128}{\pi}} \approx 40.7 \text{ cm}
\end{aligned}
$$

9 **a**
$$
\begin{aligned}
xy + ax &= b \\
x(y + a) &= b \\
x &= \frac{b}{a + y}
\end{aligned}
$$

b
$$
\begin{aligned}
\frac{a}{x} + \frac{b}{x} &= c \\
\frac{ax}{x} + \frac{bx}{x} &= cx \\
a + b &= cx \\
x &= \frac{a + b}{c}
\end{aligned}
$$

c
$$
\begin{aligned}
\frac{x}{a} &= \frac{x}{b} + 2 \\
\frac{xab}{a} &= \frac{xab}{b} + 2ab \\
bx &= ax + 2ab \\
bx - ax &= 2ab \\
x(b - a) &= 2ab \\
x &= \frac{2ab}{b - a}
\end{aligned}
$$

d $$\frac{a-dx}{d}+b=\frac{ax+d}{b}$$
$$\frac{bd(a-dx)}{d}+bd\times b=\frac{bd(ax+d)}{b}$$
$$b(a-dx)+b^2d=d(ax+d)$$
$$ab-bdx+b^2d=adx+d^2$$
$$-bdx-adx=d^2-ab-b^2d$$
$$-x(bd+ad)=-(ab+b^2d-d^2)$$
$$x=\frac{-(ab+b^2d-d^2)}{-(bd+ad)}$$
$$=\frac{ab+b^2d-d^2}{bd+ad}$$

10a $$\frac{p}{p+q}+\frac{q}{p-q}=\frac{p(p-q)+q(p+q)}{(p+q)(p-q)}$$
$$=\frac{p^2-qp+qp+q^2}{p^2-pq+pq-q^2}$$
$$=\frac{p^2+q^2}{p^2-q^2}$$

b $$\frac{1}{x}-\frac{2y}{xy-y^2}=\frac{(xy-y^2)-2xy}{x(xy-y^2)}$$
$$=\frac{-xy-y^2}{x^2y-xy^2}$$
$$=\frac{y(-x-y)}{xy(x-y)}$$
$$=\frac{-x-y}{x(x-y)}$$
$$=\frac{x+y}{x(y-x)}$$

c $$\frac{x^2+x-6}{x+1}\times\frac{2x^2+x-1}{x+3}=\frac{(x-2)(x+3)}{x+1}\times\frac{(x+1)(2x-1)}{x+3}$$
$$=(x-2)(2x-1)$$

d $$\frac{2a}{2a+b}\times\frac{2ab+b^2}{ba^2}=\frac{2a}{2a+b}\times\frac{b(2a+b)}{ba^2}$$
$$=\frac{2ab}{ba^2}$$
$$=\frac{2}{a}$$

11 Let A's age be a, B's age be b and C's age be c.

$$a=3b$$
$$b+3=3(c+3)$$
$$a+15=3(c+15)$$

Substitute for a and simplify:

$$b+3=3(c+3)$$
$$b+3=3c+9$$
$$b=3c+6 \qquad (1)$$
$$3b+15=3(c+15)$$
$$3b+15=3c+45$$
$$3b=3c+30$$
$$b=c+10 \qquad (2)$$

①=②:

$$\begin{aligned} 3c + 6 &= c + 10 \\ 3c - c &= 10 - 6 \\ 2c &= 4 \\ c &= 2 \\ b &= 3 \times 2 + 6 \\ &= 12 \\ a &= 3 \times 12 \\ &= 36 \end{aligned}$$

A, B and C are 36, 12 and 2 years old respectively.

12a Simplify the first equation:

$$\begin{aligned} a - 5 &= \frac{1}{7}(b + 3) \\ 7(a - 5) &= b + 3 \\ 7a - 35 &= b + 3 \\ 7a - b &= 38 \end{aligned}$$

Simplify the second equation:

$$\begin{aligned} b - 12 &= \frac{1}{5}(4a - 2) \\ 5(b - 12) &= 4a - 2 \\ 5b - 60 &= 4a - 2 \\ -4a + 5b &= 58 \end{aligned}$$

Multiply the first equation by 5, and add the second equation.

$35a - 5b = 190$ ①

$-4a + 5b = 58$ ②

①+②:

$$\begin{aligned} 31a &= 248 \\ a &= 8 \end{aligned}$$

Substitute into the first equation:

$$\begin{aligned} 7 \times 8 - b &= 38 \\ 56 - b &= 38 \\ b &= 56 - 38 = 18 \end{aligned}$$

b Multiply the first equation by p.

$$(p - q)x + (p + q)y = (p + q^2)$$

$$p(p - q)x + p(p + q)y = p(p + q^2) \quad ①$$

Multiply the second by $(p + q)$.

$$qx - py = q^2 - pq$$

$$q(p + q)x - p(p + q) = (p + q)(q^2 - pq) \quad ②$$

①+②:

$$\begin{aligned} (p(p - q) + q(p + q))x &= p(p + q)^2 + (p + q)(q^2 - pq) \\ (p^2 - pq + pq + q^2)x &= p(p^2 + 2pq + q^2) + pq^2 - p^2q + q^3 - pq^2 \\ (p^2 + q^2)x &= p^3 + 2p^2q + pq^2 - p^2q + q^3 \\ &= p^3 + p^2q + pq^2 + q^3 \\ &= p^2(p + q) + q^2(p + q) \\ &= (p + q)(p^2 + q^2) \\ x &= p + q \end{aligned}$$

Substitute into the second equation, factorising the right side.

$$
\begin{aligned}
q(p+q)-py &= q^2-pq \\
pq+q^2-py &= q^2-pq \\
-py &= q^2-pq-pq-q^2 \\
-py &= -2pq \\
y &= \frac{-2pq}{-p} \\
&= 2q
\end{aligned}
$$

13 $\text{Time} = \dfrac{\text{distance}}{\text{speed}}$

$\text{Remainder} = 50 - 7 - 7 = 36 \text{ km}$

$$
\begin{aligned}
\frac{7}{x}+\frac{7}{4x}+\frac{36}{6x+3} &= 4 \\
\frac{7}{x}+\frac{7}{4x}+\frac{12}{2x+1} &= 4 \\
(4x(2x+1)) \times \left(\frac{7}{x}+\frac{7}{4x}+\frac{12}{2x+1}\right) &= 4 \times 4x(2x+1) \\
28(2x+1)+7(2x+1)+48x &= 16x(2x+1) \\
56x+28+14x+7+48x &= 32x^2+16x \\
56x+28+14x+7+48x-32x^2-16x &= 0 \\
-32x^2+102x+35 &= 0 \\
32x^2-102x-35 &= 0 \\
(2x-7)(16x+5) &= 0 \\
2x-7 = 0 \text{ or } 16x+5 &= 0 \\
x > 0, \text{ so } 2x-7 &= 0 \\
x &= 3.5
\end{aligned}
$$

14a

$$
\begin{aligned}
2n^2 \times 6nk^2 \div 3n &= \frac{2n^2 \times 6nk^2}{3n} \\
&= \frac{12n^3k^2}{3n} \\
&= 4n^2k^2
\end{aligned}
$$

b

$$
\begin{aligned}
\frac{8c^2x^3y}{6a^2b^3c^3} \div \frac{\frac{1}{2}xy}{15abc^2} &= \frac{8c^2x^3y}{6a^2b^3c^3} \div \frac{xy}{30abc^2} \\
&= \frac{8c^2x^3y}{6a^2b^3c^3} \times \frac{30abc^2}{xy} \\
&= \frac{240abc^4x^3y}{6a^2b^3c^3xy} \\
&= \frac{40cx^2}{ab^2}
\end{aligned}
$$

15

$$
\begin{aligned}
\frac{x+5}{15}-\frac{x-5}{10} &= 1+\frac{2x}{15} \\
\frac{30(x+5)}{15}-\frac{30(x-5)}{10} &= 30 \times \left(1+\frac{2x}{15}\right) \\
2(x+5)-3(x-5) &= 30+4x \\
2x+10-3x+15 &= 30+4x \\
2x-3x-4x &= 30-10-15 \\
-5x &= 5 \\
x &= -1
\end{aligned
}
$$

Solutions to multiple-choice questions

1 **A**

$$
\begin{aligned}
5x+2y &= 0 \\
2y &= -5x
\end{aligned}
$$

$$\frac{y}{x} = -\frac{5}{2}$$

2 **A** Multiply both sides of the second equation by 2.

$$3x + 2y = 36 \quad (1)$$
$$6x - 2y = 24 \quad (2)$$

$(1) + (2)$:

$$\begin{aligned} 9x &= 60 \\ x &= \frac{20}{3} \\ 3 \times \frac{20}{3} - y &= 12 \\ 20 - y &= 12 \\ y &= 8 \end{aligned}$$

3 **C**
$$\begin{aligned} t - 9 &= 3t - 17 \\ t - 3t &= 9 - 17 \\ -2t &= -8 \\ t &= 4 \end{aligned}$$

4 **A**
$$\begin{aligned} m &= \frac{n-p}{n+p} \\ m(n+p) &= n - p \\ mn + mp &= n - p \\ mp + p &= n - mn \\ p(m+1) &= n(1-m) \\ p &= \frac{n(1-m)}{1+m} \end{aligned}$$

5 **B**
$$\begin{aligned} \frac{3}{x-3} - \frac{2}{x+3} &= \frac{3(x+3) - 2(x-3)}{(x-3)(x+3)} \\ &= \frac{3x + 9 - 2x + 6}{x^2 - 9} \\ &= \frac{x + 15}{x^2 - 9} \end{aligned}$$

6 **E**
$$\begin{aligned} 9x^2y^3 \div 15(xy)^3 &= \frac{9x^2y^3}{15(xy)^3} \\ &= \frac{9x^2y^3}{15x^3y^3} \\ &= \frac{9}{15x} \\ &= \frac{3}{5x} \end{aligned}$$

7 **B**
$$\begin{aligned} V &= \frac{1}{3}h(l+w) \\ 3V &= h(l+w) \\ 3V &= hl + hw \\ hl &= 3V - hw \\ l &= \frac{3V - hw}{h} \\ &= \frac{3V}{h} - w \end{aligned}$$

8 **B**
$$\begin{aligned}\frac{(3x^2y^3)^2}{2x^2y} &= \frac{9x^4y^6}{2x^2y}\\ &= \frac{9x^2y^5}{2}\\ &= \frac{9}{2}x^2y^5\end{aligned}$$

9 **B**
$$\begin{aligned}Y &= 80\% \times Z = \frac{4}{5}Z\\ X &= 150\% \times Y = \frac{3}{2}Y\\ &= \frac{3}{2} \times \frac{4Z}{5}\\ &= \frac{12Z}{10}\\ &= 1.2Z\\ &= 20\% \text{ greater than } Z\end{aligned}$$

10 **B** Let the other number be n.
$$\begin{aligned}\frac{x+n}{2} &= 5x+4\\ x+n &= 2(5x+4)\\ &= 10x+8\\ n &= 10x+8-x\\ &= 9x+8\end{aligned}$$

Solutions to extended-response questions

1 Jack cycles $10x$ km.
Benny drives $40x$ km.

a
$$\begin{aligned}\text{Distance} &= \text{speed} \times \text{time}\\ \therefore \quad \text{time} &= \frac{\text{distance}}{\text{speed}}\\ \therefore \quad \text{time taken by Jack} &= \frac{10x}{8}\\ &= \frac{5x}{4} \text{ hours}\end{aligned}$$

b
$$\begin{aligned}\text{Time taken by Benny} &= \frac{40x}{70}\\ &= \frac{4x}{7} \text{ hours}\end{aligned}$$

c
$$\begin{aligned}\text{Jack's time--Benny's time} &= \frac{5x}{4} - \frac{4x}{7}\\ &= \frac{(35-16)x}{7}\\ &= \frac{19x}{28} \text{ hours}\end{aligned}$$

d **i**
$$\begin{aligned}\text{If the difference is 30 mins} &= \frac{1}{2} \text{ hour}\\ \text{then } \frac{19x}{28} &= \frac{1}{2}\\ \therefore \quad x &= \frac{14}{19}\\ &= 0.737 \text{ (correct to three decimal places)}\end{aligned}$$

ii Distance for Jack $= 10 \times \frac{14}{19}$

$$= \frac{140}{19}$$

$$= 7 \text{ km (correct to the nearest km)}$$

Distance for Benny $= 40 \times \frac{14}{19}$

$$= \frac{560}{19}$$

$$= 29 \text{ km (correct to the nearest km)}$$

2 a Dinghy is filling with water at a rate of $27\,000 - 9\,000 = 18\,000 \text{ cm}^3$ per minute.

b After t minutes there are $18\,000t \text{ cm}^3$ water in the dinghy, i.e. $V = 18\,000t$

c $V = \pi r^2 h$ is the formula for the volume of a cylinder

$$\therefore \quad h = \frac{V}{\pi r^2} = \frac{18\,000t}{\pi r^2}$$

The radius of this cylinder is 40 cm

$$\therefore \quad h = \frac{18\,000t}{1600\pi} \qquad = \frac{45t}{4\pi}$$

i.e. the height h cm water at time t is given by $h = \dfrac{45t}{4\pi}$

d When $t = 9,\ h = \dfrac{45 \times 9}{4\pi}$

$\approx 32.228\ldots$

The dinghy has filled with water, before $t = 9$, i.e. Sam is rescued after the dinghy completely filled with water.

3 a Let Thomas have a cards. Therefore Henry has $\frac{5a}{6}$ cards, George has $\frac{3a}{2}$ cards, Sally has $(a - 18)$ cards and Zeb has $\frac{a}{3}$ cards.

b $\dfrac{3a}{2} + a - 18 + \dfrac{a}{3} = a + \dfrac{5a}{6} + 6$

c $\therefore \quad 9a + 6a - 108 + 2a = 6a + 5a + 36$

$$\therefore \quad 6a = 144$$

$$\therefore \quad a = 24$$

Thomas has 24 cards, Henry has 20 cards, George has 36 cards, Sally has 6 cards and Zeb has 8 cards.

4 a $F = \dfrac{6.67 \times 10^{-11} \times 200 \times 200}{12^2}$

$$= 1.852\ldots \times 10^{-8}$$

$$= 1.9 \times 10^{-8} \text{ N (correct to two significant figures)}$$

b $m_1 = \dfrac{Fr^2}{m_2 \times 6.67 \times 10^{-11}}$

$$= \frac{Fr^2 \times 10^{11}}{6.67 m_2}$$

c If $F = 2.4 \times 10^4$

$$r = 6.4 \times 10^6$$

and $m_2 = 1500$

$$m_1 = \frac{2.4 \times 10^4 \times (6.4 \times 10^6)^2 \times 10^{11}}{6.67 \times 1500}$$

$$= 9.8254\ldots \times 10^{24}$$

The mass of the earth is 9.8×10^{24} kg (correct to two significant figures).

5 a $V = 3 \times 10^3 \times 6 \times 10^3 \times d$

$$= 18 \times 10^6 d$$

$$= 1.8 \times 10^5 d$$

b When $d = 30,\ V = 18 \times 10^6 \times 30$

$$= 540\ 000\ 000$$

$$= 5.4 \times 10^8$$

The volume of the reservoir is 5.4×10^8 m^3.

c

$$E = kVh$$

$$1.06 \times 10^{15} = k \times 200 \times 5.4 \times 10^8$$

$$k = \frac{1.06 \times 10^{15}}{200 \times 5.4 \times 10^8}$$

$$= 9.81\ldots \times 10^3$$

$k = 9.81 \times 10^3$ correct to three significant figures.

d $E = (9.81 \times 10^3) \times 5.4 \times 10^8 \times 250$

$= 1.325 \times 10^{15}$ correct to four significant figures.

The amount of energy produced is 1.325×10^{15} J.

e Let t be the time in seconds.

$$5.2 \times t = 5.4 \times 10^8$$

$$t = 103.846\ 153\ 8$$

$$\therefore \quad \text{number of days} = 103.846\ 153\ 8 \div (24 \times 60 \times 60)$$

$$= 1201.92\ldots$$

The station could operate for approximately 1202 days.

CAS calculator techniques for Question 5

5 b Calculations involving scientific notation and significant figures can be accomplished with the aid of a graphics calculator.

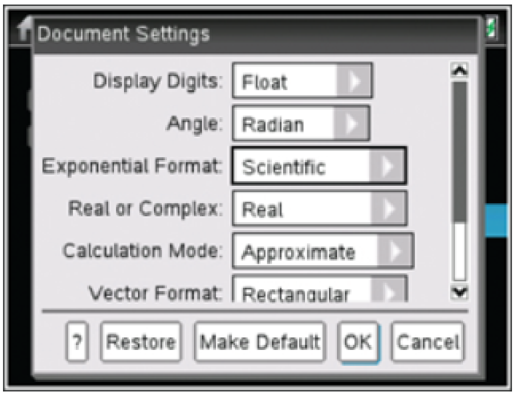
When $d = 30,\ V = 18 \times 10^6 \times 30$

$$= 540\ 000\ 000$$

This calculation can be completed as shown here.

T1: Press c → **5**: **Settings** → **2**: **Document**

Settings and change the Exponential Format to Scientific. Click on Make Default.

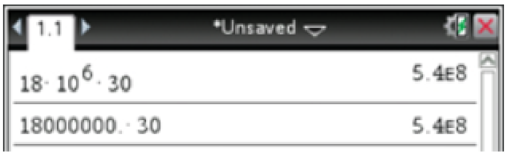


Return to the Calculator application.

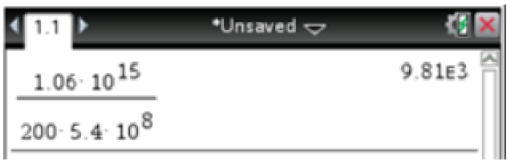
Type $\mathbf{18} \times \mathbf{10\hat{}6} \times \mathbf{30}$ or $\mathbf{18i6} \times \mathbf{30}$

CP: In the Main application tap ◯ → **Basic Format**

Change the Number Format to Sci 2 Type $\mathbf{18} \times \mathbf{10\hat{}6} \times \mathbf{30}$



c **T1**: Press c → **5: Settings** → **2: Document Settings** and change the Display Digits to Float 3. Click on Make Default.

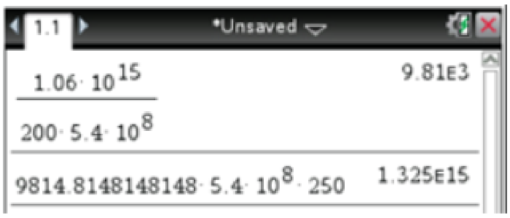


Return to the home screen and press and complete as shown.

CP: tap ◯ → **Basic Format**

Change the Number Format to Sci3 Complete calculation as shown

d The calculation is as shown. **T1**: Display Digits is Float 4 **CP**: Number Format is Sci 4 Simply type $\times \mathbf{5.4} \times \mathbf{10\hat{}8} \times \mathbf{25}$



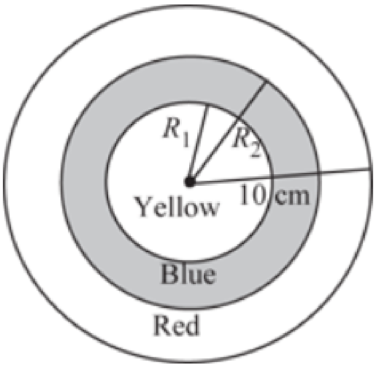
6 Let R_1 cm and R_2 cm be the radii of the inner circles.

$$\begin{aligned}
\therefore \quad & \text{Yellow area} = \pi R_1^2 \\
& \text{Blue area} = \pi R_2^2 - \pi R_1^2 \\
& \text{Red area} = 100\pi - \pi R_2^2 \\
\therefore \quad & 100\pi - \pi R_2^2 = \pi R_2^2 - \pi R_1^2 = \pi R_1^2 \\
\text{Firstly,} \quad & \pi R_2^2 - \pi R_1^2 = \pi R_1^2 \\
\text{implies} \quad & R_2^2 = 2R_1^2 \qquad \text{①} \\
\text{and} \quad & 100\pi - \pi R_2^2 = \pi R_2^2 - \pi R_1^2 \\
\text{implies} \quad & 100 = 2R_2^2 - R_1^2 \qquad \text{②}
\end{aligned}$$

Substitute from ① in ②

$$\begin{aligned}
\therefore \quad & 100 = 4R_1^2 - R_1^2 \\
& 100 = 3R_1^2 \\
\text{and} \quad & R_1 = \frac{10}{\sqrt{3}} \\
& = \frac{10\sqrt{3}}{3} \qquad \left(\text{Note}: \ R_2^2 = \frac{200}{3}\right)
\end{aligned}$$

The radius of the innermost circle is $\frac{10\sqrt{3}}{3}$ cm.



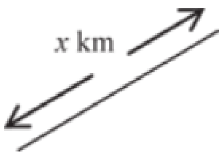
7 If $C = F$,

$$F = \frac{5}{9}(F - 32)$$
$$9F = 5F - 160$$
$$\therefore 4F = -160$$
$$\therefore F = -40$$

Therefore $-40°\text{F} = -40°\text{C}$.

8 Let x km be the length of the slope.

$$\therefore \quad \text{time to go up} = \frac{x}{15}$$
$$\therefore \quad \text{time to go down} = \frac{x}{40}$$
$$\therefore \quad \text{total time} = \frac{x}{15} + \frac{x}{40} = \frac{11x}{120}$$
$$\therefore \quad \text{average speed} = \frac{\text{total distance}}{\text{total time}} = 2x \div \frac{11x}{120} = 2x \times \frac{120}{11x} = \frac{240}{11} \approx 21.82 \text{ km/h}$$



9 $1 \text{ litre} = 1000 \text{ cm}^3$

a $$\text{Volume} = \text{Volume of cylinder} + \text{Volume of hemisphere} = \pi r^2 h + \frac{2}{3}\pi r^3$$

It is known that $r + h = 20$

$$\therefore \quad h = 20 - r$$

b i $$\text{Volume} = \pi r^2(20 - r) + \frac{2}{3}\pi r^3 = 20\pi r^2 - \pi r^3 + \frac{2}{3}\pi r^3 = 20\pi r^2 - \frac{\pi}{3}r^3$$

ii If $\text{Volume} = 2000 \text{ cm}^3$

then $20\pi r^2 - \frac{\pi}{3}r^3 = 2000$

Use a CAS calculator to solve this equation for r, given that $0 < r < 20$. This gives$r = 5.943999\cdots$

$$\begin{aligned}\text{Therefore } h &= 20 - r\\ &= 20 - 5.943\,99\ldots\\ &= 14.056\,001\ldots\end{aligned}$$

The volume is two litres when $r = 5.94$ and $h = 14.06$, correct to two decimal places.

10a Let x and y be the amount of liquid (in cm^3) taken from bottles A and B respectively.
Since the third bottle has a capacity of $1000\ \mathrm{cm}^3$,

$$x + y = 1000 \quad (1)$$

$$\text{Now} \qquad x = \frac{2}{3}x \text{ wine} + \frac{1}{3}x \text{ water}$$

$$\text{and} \qquad y = \frac{1}{6}y \text{ wine} + \frac{5}{6}y \text{ water}$$

$$\therefore \quad \frac{2}{3}x + \frac{1}{6}y = \frac{1}{3}x + \frac{5}{6}y \text{ since the proportion of wine and water must be the same.}$$

$$\therefore \quad 4x + y = 2x + 5y$$

$$\therefore \quad 2x = 4y$$

$$\therefore \quad x = 2y$$

$$\text{From (2)} \quad 2y + y = 1000$$

$$\therefore \quad y = \frac{1000}{3} \text{ and } x = \frac{2000}{3}$$

Therefore, $\frac{2000}{3}\ \mathrm{cm}^3$ and $\frac{1000}{3}\ \mathrm{cm}^3$ must be taken from bottles A and B respectively so that the third bottle will have equal amounts of wine and water, i.e. $\frac{2}{3}L$ from A and $\frac{1}{3}L$ from B

b

$$x + y = 1000 \quad (1)$$

$$\frac{1}{3}x + \frac{3}{4}y = \frac{2}{3}x + \frac{1}{4}y$$

$$\therefore \quad 4x + 9y = 8x + 3y$$

$$\therefore \quad 6y = 4x$$

$$\therefore \quad x = \frac{3}{2}y \quad (2)$$

$$\text{From (1)} \quad \frac{3}{2}y + y = 1000$$

$$\begin{aligned}\therefore \quad y &= \frac{2}{5} \times 1000\\ &= 400\end{aligned}$$

$$\therefore \quad x = 600$$

Therefore, $600\ \mathrm{cm}^3$ and $400\ \mathrm{cm}^3$ must be taken from bottles A and B respectively so that the third bottle will have equal amounts of wine and water, i.e. $600\ \mathrm{mL}$ from A and $400\ \mathrm{mL}$ from B

c

$$x + y = 1000 \quad (1)$$

$$\frac{m}{m+n}x + \frac{p}{p+q}y = \frac{n}{m+n}x + \frac{q}{p+q}y$$

$$\therefore \; m(p+q)x + p(m+n)y = n(p+q)x + q(m+n)y$$

$$\therefore \; (m(p+q) - n(p+q))x = (q(m+n) - p(m+n))y$$

$$\therefore \; (m-n)(p+q)x = (q-p)(m+n)y$$

$$\therefore \; x = \frac{(m+n)(q-p)}{(m-n)(p+q)}y, \; m \neq n, p \neq q \quad (2)$$

From (1)

$$\frac{(m+n)(q-p)}{(m-n)(p+q)}y + y = 1000$$

$$\therefore \; \frac{(m+n)(q-p) + (m-n)(p+q)}{(m-n)(p+q)}y = 1000$$

$$\therefore \; \frac{mq - mp + nq - np + mp + mq - np - nq}{(m-n)(p+q)}y = 1000$$

$$\therefore \; \frac{2(mq-np)}{(m-n)(p+q)}y = 1000$$

$$\therefore \; y = \frac{500(m-n)(p+q)}{mq-np}, \; mq \neq np$$

From (1) $x = \dfrac{(m+n)(q-p)}{(m-n)(p+q)} \times \dfrac{500(m-n)(p+q)}{mq-np}$

$$= \frac{500(m+n)(q-p)}{mq-np}, \; \frac{n}{q} \neq \frac{q}{p}$$

Therefore, $\dfrac{500(m+n)(q-p)}{mq-np}$ cm^3 and $\dfrac{500(m-n)(p+q)}{mq-np}$ cm^3 must be taken from bottles A and B respectively so that the third bottle will have equal amounts of wine and water. In litres this is $\dfrac{(m+n)(q-p)}{2(mq-np)}$ litres from A and $\dfrac{(m-n)(p+q)}{2(mq-np)}$ litres from B. Also note that $\dfrac{n}{m} \geq 1$ and $\dfrac{q}{p} \leq 1$ or $\dfrac{n}{m} \leq 1$ and $\dfrac{q}{p} \geq 1$.

11a

$$\frac{20-h}{20} = \frac{r}{10}$$

$$\therefore \; 10(20-h) = 20r$$

$$\therefore \; 200 - 10h = 20r$$

$$\therefore \; 20 - h = 2r$$

$$\therefore \; h = 20 - 2r$$

$$= 2(10 - r)$$

b $V = \pi r^2 h$

$$= 2\pi r^2(10 - r)$$

c Use CAS calculator to solve the equation $2\pi x^2(10 - r) = 500$, given that $0 < r < 10$.

This gives $r = 3.49857\ldots$ or $r = 9.02244\ldots$

When $r = 3.498\,57\ldots$, $h = 2(10 - 3.498\,57\ldots)$
$= 13.002\,85\ldots$

; When $r = 9.022\,44\ldots$, $h = 2(10 - 9.022\,44\ldots)$
$= 1.955\,11\ldots$

Therefore the volume of the cylinder is $500 \mathrm{~cm}^3$ when $r = 3.50$ and $h = 13.00$ or when $r = 9.02$ and $h = 1.96$, correct to two decimal places.